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Susceptibilities of the antiferromagnet REM model

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Abstract. The linear and non-linear susceptibilities of the two sub-lattices Random Energy Model (REM) allowing antiferromagnetic order is studied as a function of the external field (h) and temperature (T). Due to the competition between external field and the internal exchange field acting on the spins there is a drastic change of the system's behavior as the parameters (h, T) are varied. The behavior of the susceptibilities in low and high fields is very different in that the latter may grow as the temperature decreases. Moreover, the critical region undergoes a substantial enlargement as the external field increases.

PACS. 75.10.Nr Spin-glass and other random models – 75.40.Cx Static properties (order parameter, static susceptibility, heat capacities, critical exponents, etc.) – 75.50.Lk Spin glasses and other random magnets

1 Introduction

Disordered magnetic systems have been intensely studied both theoretically and experimentally over the past decades. Proeminent among these systems one finds the spin glasses and diluted antiferromagnets in a field (random field problem) [1]. There is a wealth of unsolved questions both from the viewpoint experimental and theoretical, despite the huge progress in the field. Simple models retaining the fundamental aspects of the problem and which can be worked out thoughrouly in such a scenario are a blessing welcome by workers in the field. The canonical model for spin glasses is the Edwards-Anderson model [2] which led to the well known Sherrington-Kirkpatrick (SK) mean field model [3]. The solution of the SK model is rather intricate [1] for the low temperature phase is highly degenerate and without any obvious symmetry among the possible states in addition to the presence of broken ergodicity.

A simple model, yet retaining all crucial features of the SK model, was introduced long ago by Derrida [4]. This is the Random Energy Model (REM) which considers the possible energy states of the system as random variables and is equivalent to a multispin interacting Ising model in the limit when the number of interacting units tends to infinity. The model is exactly solvable with or without replicas and was termed "the simplest spin glass" [5]. It has been applied in areas distinct from magnetic systems such as in biophysics and related problems [6,7,8]. In the present work the REM is generalized for two-sublattices systems having an uniform exchange antiferromagnetic interaction between the sublattices. One finds [9], as expected, several phases: paramagnetic, antiferromagnetic, spin glass, and mixed spin glass-antiferromagnetic. All these phases persist in the presence of an external field and due to the various competing fields acting on the spins there may arise a rich variety of the system's response functions. The linear and higher order susceptibilities are worked out and exhibit a behavior similar to recent experimental results [10]. In particular, the low temperature behavior of the susceptibility depends strongly on the intensity of the applied field: it may increases as T decreases and even display a maximum at low T. A related analysis for the SK model has been presented before [11], though lacking a full numerical solution of the equations. Here we present the numerical solution of the antiferromagnetic REM model identifying all the main qualitative aspects of the model and pointing out the substantial increase of the critical region as h increases.

2 The model

The antiferromagnetic REM model may be defined through the following multispin interacting Hamiltonian

$$\mathcal{H} = -\sum_{1\le i_1<i_2\cdots<i_p}\ \sum_{1\le j_1<j_2\cdots<j_p} \times J_{i_1 i_2\ldots i_p j_1 j_2\ldots j_p}\, S_{i_1}S_{i_2}\ldots S_{i_p}\sigma_{j_1}\sigma_{j_2}\ldots\sigma_{j_p} + \frac{J_0}{N}\sum_{i,j} S_i\sigma_j - h\sum_i (S_i+\sigma_i) \quad (1)$$

where the sets $\{S_i = \pm 1\}$, $\{\sigma_i = \pm 1\}$, $i = 1, 2, \ldots, N$ are Ising spin variables on distinct sublattices, $J_0 > 0$ is a

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uniform antiferromagnetic pair interaction among spins S and σ, h is an external field and the $J_{i_1 i_2 \ldots j_1 j_2 \ldots j_p}$ are random exchange interactions having a Gaussian probability distribution,

$$P\left(J_{i_1 i_2 \ldots j_p}\right) = \sqrt{\frac{N^{2p-1}}{\pi J^2 (p!)^2}} \exp\left\{-\frac{\left(J_{i_1 i_2 \ldots j_p}\right)^2 N^{2p-1}}{J^2 (p!)^2}\right\} \tag{2}$$

where the N scaling ensures a nontrivial thermodynamic limit and at the end of calculation the limit $p \to \infty$ is taken. The SK result is obtained through the substitution $p = 1$ and $J \to \sqrt{2}J$ (from now on we take $J = 1$). Following standard procedure [4,5], the free energy per spin within the replica approach is given by

$$\begin{aligned} f = -\frac{\beta}{8} + \lim_{n \to 0} \frac{1}{2n\beta} & \left\{ \frac{1}{2} \sum_{s=1}^{2} \sum_{\alpha=1}^{n} \sum_{\beta \neq \alpha}^{n} \lambda_s^{\alpha\beta} \mathcal{Q}_s^{\alpha\beta} \right. \\ & + \sum_{s=1}^{2} \sum_{\alpha=1}^{n} \Lambda_s^{\alpha} m_s^{\alpha} - \beta h \sum_{s=1}^{2} \sum_{\alpha=1}^{n} m_s^{\alpha} \\ & - \frac{\beta^2}{4} \sum_{\alpha=1}^{n} \sum_{\beta \neq \alpha} \left(\mathcal{Q}_1^{\alpha\beta} \mathcal{Q}_2^{\alpha\beta}\right)^p + J_0 \sum_{\alpha=1}^{n} m_1^{\alpha} m_2^{\alpha} \\ & - \ln\, Tr\, \exp\left[\frac{1}{2} \sum_{\alpha=1}^{n} \sum_{\beta \neq \alpha}^{n} \left(\lambda_1^{\alpha\beta} \sigma^{\alpha} \sigma^{\beta} + \lambda_2^{\alpha\beta} S^{\alpha} S^{\beta}\right) \right. \\ & \left. \left. + \sum_{\alpha=1}^{n} \left(\Gamma_1^{\alpha} \sigma^{\alpha} + \Gamma_2^{\alpha} S^{\alpha}\right) + h \sum_{\alpha=1}^{n} \left(\sigma^{\alpha} + S^{\alpha}\right) \right] \right\} \end{aligned} \tag{3}$$

where α, $\beta = 1, 2, ..., n$, are replica indices and $\beta = 1/T$ (taking $k_B = 1$); $\lambda_s^{\alpha\beta}$, $\mathcal{Q}_s^{\alpha\beta}$, Γ_s^{α}, m_s^{α} $(s = 1, 2)$ are variational parameters and the expression in braces is to be evaluated at the dominant saddle point, hence the equations

$$\begin{aligned} &\mathcal{Q}_1^{\alpha\beta} = \langle \sigma^{\alpha} \sigma^{\beta} \rangle; \qquad m_1^{\alpha} = \langle \sigma^{\alpha} \rangle \\ &\Gamma_1^{\alpha} = -\beta J_0 m_2^{\alpha}; \quad \lambda_1^{\alpha\beta} = \frac{\beta^2 p}{4} \left(\mathcal{Q}_1^{\alpha\beta}\right)^{p-1} \left(\mathcal{Q}_2^{\alpha\beta}\right)^p \end{aligned} \tag{4}$$

and similar equations with the interchange $1 \to 2$, $\sigma \to S$, the averages $\langle \cdots \rangle$ are calculated using the effective Hamiltonian defined by the argument of the exponential function in equation (3). The set of equations (4) has a replica symmetric solution $m_1^{\alpha} = m_1$, $m_2^{\alpha} = m_2$, $\mathcal{Q}_1^{\alpha\beta} = \mathcal{Q}_1 = m_1^2$, $\mathcal{Q}_2^{\alpha\beta} = \mathcal{Q}_2 = m_2^2$, with $\lambda_1^{\alpha\beta} = \lambda_2^{\alpha\beta} = 0$ as $p \to \infty$. The replica symmetric free energy becomes

$$\begin{aligned} f = -\frac{\beta}{8} - \frac{J_0 m_1 m_2}{4} - \frac{1}{4\beta} \ln\, &\{4 \cosh\left[\beta(h - J_0 m_1)\right] \\ &\times \cosh\left[\beta(h - J_0 m_2)\right]\} \end{aligned} \tag{5}$$

with

$$\begin{aligned} m_1 &= \text{tgh}\left[\beta(h - J_0 m_2)\right] \\ m_2 &= \text{tgh}\left[\beta(h - J_0 m_1)\right]. \end{aligned} \tag{6}$$

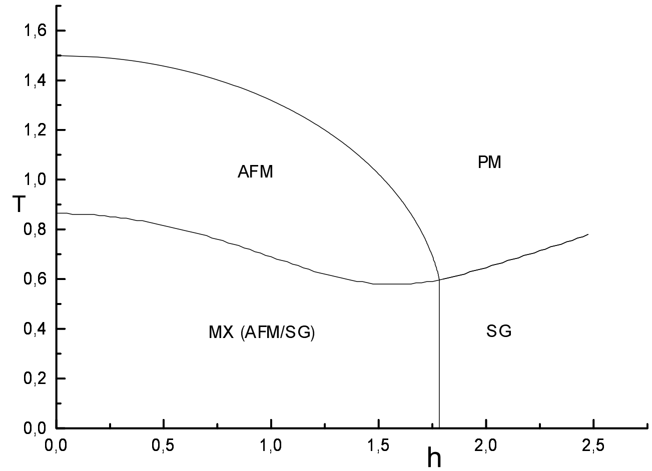

Fig. 1. The phase diagram of the antiferromagnetic REM model for $J_0 = 1.5$.

Equations (5, 6) are valid in a large part of the (T, h, J_0) parameters space, specifically in the regions where paramagnetic ($m_1 = m_2$) and antiferromagnetic ($m_1 \neq m_2$) solutions occur. However, in some regions of the parameters space the replica symmetric solution gives negative entropy and the remedy is to break replica symmetry [1,4,5,12]. For this model a one step Parisi ansatz is sufficient [5], and one finds that the system freezes completely whenever the solution to equations (5, 6) furnish zero entropy in a state of broken replica symmetry, a result obtainable with or without the replica approach. The study of the broken replica symmetry solution furnish information about the structure of the possible thermodynamic states; in this model many states exist having minimal overlap among states and maximal self-overlap [5].

3 Susceptibilities

We are here interested in working out the magnetic response functions for the model defined by equations (1, 2) when $J_0 > \sqrt{8\ln 2}$ and $p \to \infty$, *i.e.*, in that region of the parameters space where the phases paramagnetic (PM), antiferromagnetic (AFM), spin glass (SG) and mixed spin glass-antiferromagnetic (MX) may arise depending on the values of (T, h) [9]. Figure 1 shows the phase diagram of the model for $J_0 = 1.5$. Both theoretically [11,13] and experimentally [10] the following response functions are of interest:

$$\chi_K = \frac{1}{K!} \left(\frac{\partial^K M}{\partial h^K}\right)_T; \qquad K = 1, 2, 3, \cdots \tag{7}$$

where $M = M(T, h)$ is the equilibrium magnetization and we are interested in studying $\chi_K(T, h)$. For the lowest ones we have:

(i) Linear (differential) susceptibility

$$\chi_1 = (\partial M / \partial h)_T. \tag{8}$$

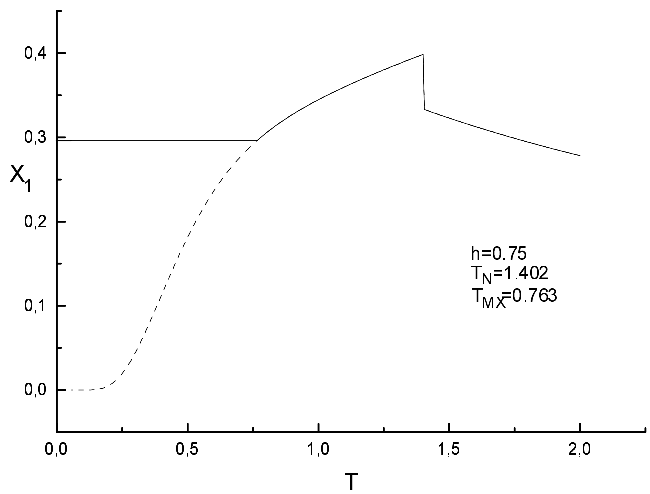

Fig. 2. First order susceptibility χ_1 for $J_0 = 1.5$, $h = 0.75$. Broken line is for pure AFM system.

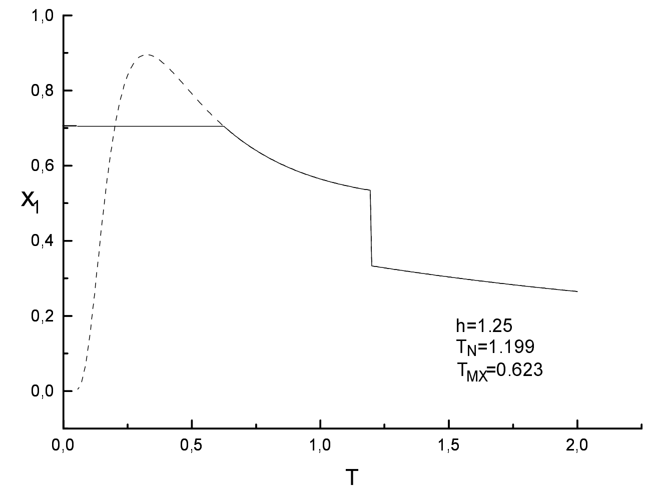

Fig. 3. First order susceptibility χ_1 for $J_0 = 1.5$, $h = 1.25$. Broken line is for pure AFM system.

From equations (5, 6) the magnetization is given by (valid for the regions PM and AFM in Fig. 1)

$$M = \frac{1}{2}(m_1 + m_2) = \frac{1}{2}\{\text{tgh}[\beta(h - J_0 m_1)] + \text{tgh}[\beta(h - J_0 m_2)]\} \quad (9)$$

where m_1, m_2 are the solution of equation (6) which minimizes f, equation (5). For the other regions, for given h, M is constant and equal to its value at the boundary AFM/MX or PM/SG [4,5,9]. This boundary is given by $T_g = T_g(h, J_0)$ to be obtained from the following equation

$$0 = \frac{\beta^2}{8} + \frac{1}{2}\ell\{5\cosh[\beta(h - J_0 m_1)]\cosh[\beta(h - J_0 m_2)]\} - \frac{1}{2}\beta m_1 \cdot (h - J_0 m_2) - \frac{1}{2}\beta m_2(h - J_0 m_1) \quad (10)$$

which delimit the regions of replica symmetric solutions from broken replica symmetry solutions [4,5]. For this model equation (10) gives the points where the entropy first reach zero when decreasing the temperature from high values. From (8, 9) χ_1 is

$$\chi_1 = \frac{1}{2}\beta\left[\frac{S_1 + S_2 - 2\beta J_0 S_1 S_2}{1 - \beta^2 J_0^2 S_1 S_2}\right] \quad (11)$$

where

$$S_1 = 1 - m_1^2, \quad S_2 = 1 - m_2^2. \quad (12)$$

For temperatures below $T_g(h, J_0)$ the system is frozen and χ_1 is constant. The numerical solution of equations (5) to (10) for $h = 0.75$ and 1.25 is shown in Figures 2 and 3. There is a clear change in the system's response as the external field is increased. At low field, Figure 2, the behavior is typical of antiferromagnetics having a decrease of χ_1 at both sides of the Néel temperature plus a freezing (mixed phase) at low T. For high fields, Figure 3, due to the "anomalous" behavior (*i.e.* non-monotonic) of the internal field $H_i = h - J_0 M$ the susceptibility keeps increasing as the temperature decreases until it assumes a constant value, which may be smaller than the value for uniform systems (dashed points in Fig. 3). It is tempting to suggest that in an experimental measurement the field cooled measurements (equilibrium) will follow the full line in Figure 3 while the zero field ones will tend to follow the dashed line, as for example in the diluted $Fe_x Zn_{1-x} F_2$ although the present model is too simple to account for the whole experimental results [10].

(ii) Non-linear susceptibility (second order)

$$\chi_2 = (\chi_2)_{St(1)} + (\chi_2)_{St(2)} \quad (13)$$

where

$$(\chi_2)_{St(i)} = \frac{1}{2}\left(\frac{\partial^2 m_i}{\partial h^2}\right); \qquad i = 1, 2 \quad (14)$$

are the staggered susceptibilities.

At very low fields $M(T, h)$ can be expanded in powers of h, time reversal symmetry requires $\chi_2(T, h = 0) = 0$ and thus $(\chi_1)_{St(1)} = -(\chi_2)_{St(2)}$. At non-zero fields this is not so and we find

$$(\chi_2)_{St(1)} = \frac{D_1 - \beta J_0 S_1 D_2}{1 - \beta^2 J_0^2 S_1 S_2} \quad (15)$$

where the S's are given by (12) and

$$D_i = -\beta m_i \left(\frac{\partial m_i}{\partial h}\right)_T \left[1 - J_0 \left(\frac{\partial m_j}{\partial h}\right)_T\right]; \; i, j (i \neq j) = 1, 2 \quad (16)$$

and a similar expression for $(\chi_2)_{St(2)}$. Both staggered susceptibilities above diverge at the Néel temperature.

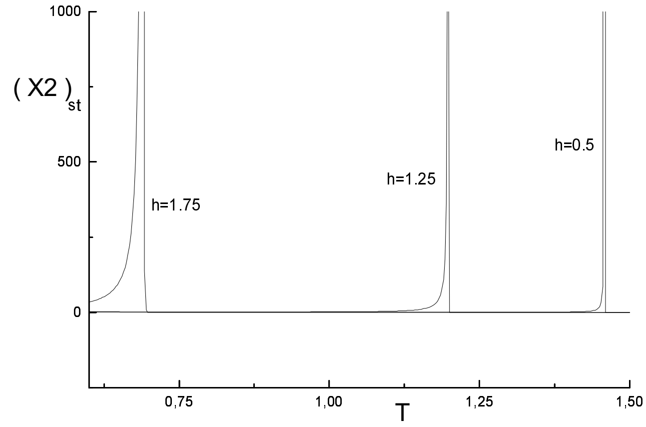

Fig. 4. Second order staggered susceptibility $(\chi_2)_{St(1)}$ for $J_0 = 1.5$. Field values are indicated.

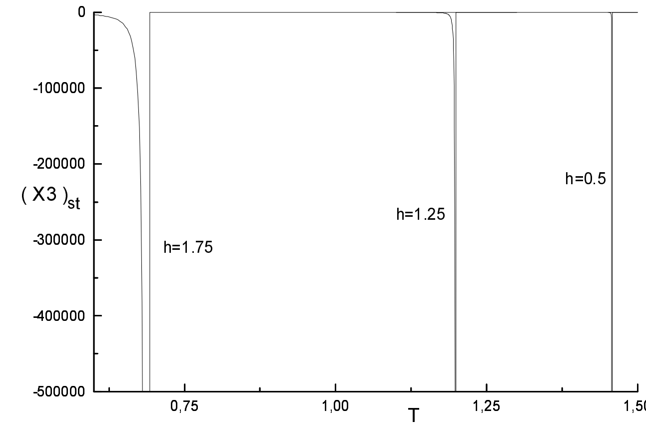

Fig. 5. Third order staggered susceptibility $(\chi_3)_{St(1)}$ for $J_0 = 1.5$. Field values are indicated.

Figure 4 shows the numerical solutions for equation (15) for various fields. It is noteworthy the huge increase of the neighborhood around T_N where $(\chi)_{St}$ is large as h increases from zero.

(iii) Non-linear susceptibility (third order)

A hallmark among the response functions of spin glasses is the negative divergence of χ_3 at the glass transition [1,13]. From (5, 6) one obtains

$$\chi_3 = (\chi_3)_{St(1)} + (\chi_3)_{St(2)} \tag{17}$$

where the terms may be written in the same form as in (14) and (16)

$$(\chi_3)_{St(i)} = \frac{1}{3!}\left(\frac{\partial^3 m_i}{\partial h^3}\right)_T; \quad i = 1, 2 \tag{18}$$

$$\left(\frac{\partial^3 m_i}{\partial h^3}\right)_T = \frac{T_i - \beta J_0 S_i T_j}{1 - \beta^2 J_0^2 S_1 S_2}; \quad i, j (i \neq j) = 1, 2. \tag{19}$$

This quantity, at zero field, diverge at the transition temperature for spin glasses and pure ferromagnets while for antiferromagnets no divergence occurs [13]. In Figure 5 we present the numerical solution for $(\chi_3)_{St(1)}$ for $h = 0.75$, 1.25 and 1.75. For finite fields there always occurs a divergence at the Néel temperature. Not shown on the scale of the figure is the cusp in χ_3 at the boundary between the phases AFM/MX. Again, notice the huge increase of the region where χ_3 is large as the field increases. In other words, for high fields ($h \simeq J_0$) χ_3 is very large in a region close to T_N whose width is comparable to T_N itself as is evident from Figure 5. En passant, we notice that an enlargement of the critical region for high fields has been continuously observed in measurements involving diluted AFM in a field [14], translated into two distincts temperatures: T_N (Néel temperature) and $T_{eq}(h)$ where irreversibility sets in for field cooled measurements. This behavior may be expressed more quantitatively through a Ginzburg-like parameter as usual.

4 Discussion

Random antiferromagnets have been intensely studied in the last decades from the viewpoint of the random field problem, spin glasses or in connection to high temperature superconductors. Here we have taken the REM model with two-sublattices as appropriated to a model which could exhibit antiferromagnetism. The behavior of the susceptibility, equation (7), as a function of the external field has been shown to exhibit distinct behavior at low and high fields, Figures 2 and 3. The region of fields where only the phases PM and SG can exist has not been shown for it has been considered previously [4,5]. The non-analytic model's behavior has been studied through the second and third order staggered susceptibilities equations (14, 18). In a field these quantities diverge at the Néel temperature and as h increases they are very large in a substantial temperature interval close to T_N. Critical behavior is thus amplified in this case, and has been observed in measurements involving diluted AFM in a field [14] where the region of critical slowing down increases as h increases, in accord to Figures 4 and 5. Physically, all this behavior stems from the two-sublattice structure with opposing staggered magnetizations in the presence of an external uniform field. In the present work we have exploited the numerical solution of the equations reserving a more analytical treatment as well as obvious extensions to future work.

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